(Basic!) Antiderivatives

We know that if $f(x) = 3x^2 + 2x - 4$ then $f'(x) = 6x + 2$.

What if we are given a function and asked whether it is the derivative of some function and (if so) what function that might be?

Example: Suppose $y' = 5x + 1$, what can we say about y?

$$y = \frac{5}{2}x^2 + x \quad + C$$

⟸ How did I know this?

Let's think about differentiating for a moment . . .

$y = x^{10} \longrightarrow y' = 10x^9$

So if $y' = x^7 \longrightarrow y = \frac{1}{8}x^8$

↳ 1st guess

Next:

$y = e^{3x} \longrightarrow y' = 3e^{3x} = 3y$

So if $y' = e^{-2x} \longrightarrow y = -\frac{1}{2}e^{-2x}$

↳ 1st guess

$y = \frac{1}{8}x^8 \rightarrow$ what about $y = \frac{1}{8}x^8 - 3$?

↳ 2nd guess

↳ An antiderivative

$y = \frac{1}{8}x^8 + C$

↳ The general antiderivative

If $F(x) = x^n$, then $f(x) = \frac{x^{n+1}}{n+1} + C$

is *the general antiderivative* for F.

$g(x) = \frac{x^{n+1}}{n+1} - 3$

is *a particular antiderivative* for F.

If $F(x) = e^{ax}$, then $f(x) = \frac{1}{a}e^{ax} + C$